

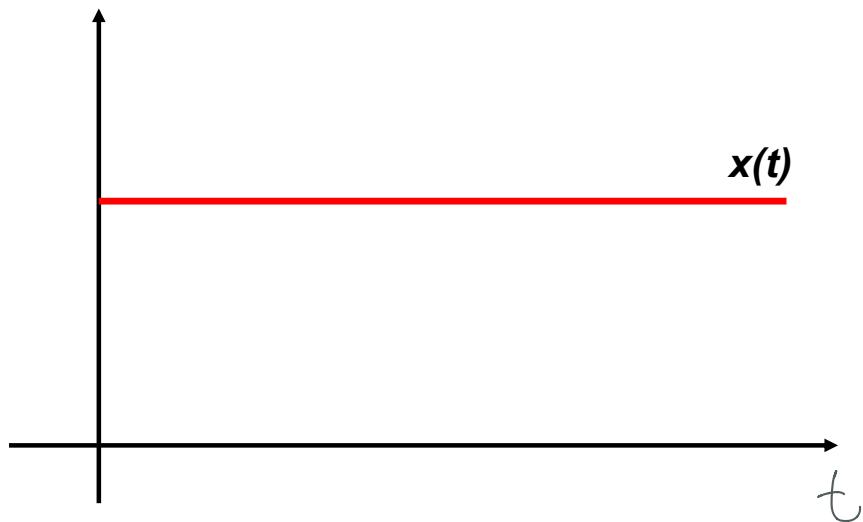


Repaso conceptos básicos y análisis



Sistemas con excitaciones continuas

$$\frac{dx}{dt} = 0$$

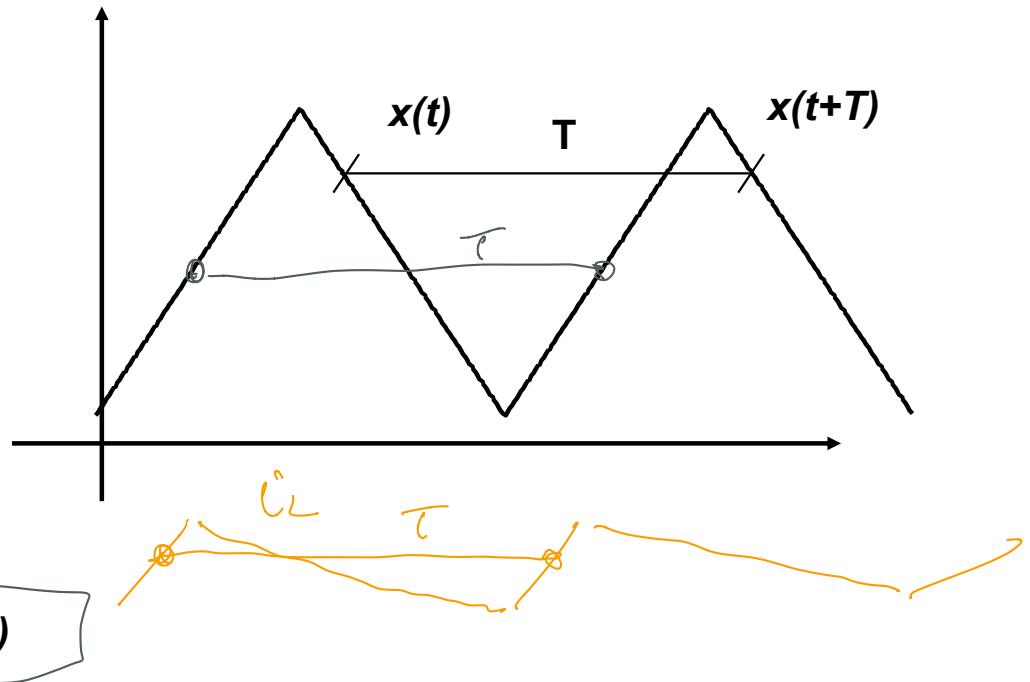




Régimen Permanente

Sistemas con excitación periódica

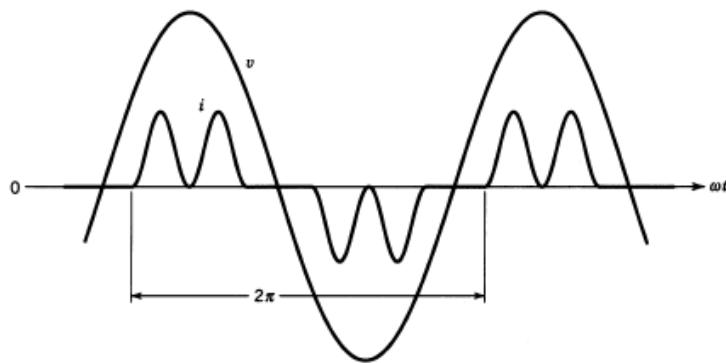
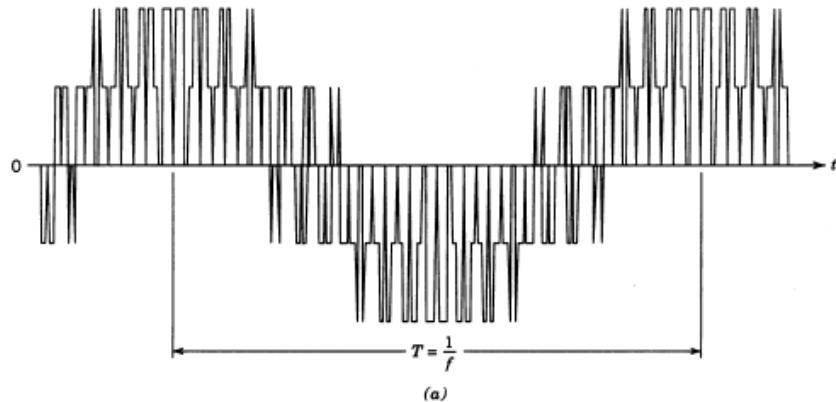
$$\cancel{\frac{dx}{dt} = 0}$$





Régimen Permanente

Ejemplo en electrónica de potencia





Condición de régimen permanente

$$E_L = \frac{1}{2} L i^2$$

Balance de flujo en la bobina

$$I_L = \text{cte}$$

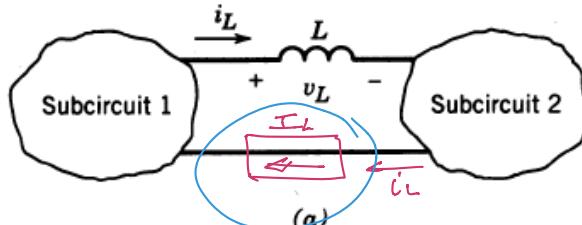
$$\Delta i \rightarrow 0$$

$$\text{Si } i = \frac{v}{L} \cdot \Delta t$$

$$v = L \frac{di}{dt}$$

$L \uparrow \uparrow \Rightarrow \Delta i \rightarrow 0 \Rightarrow$

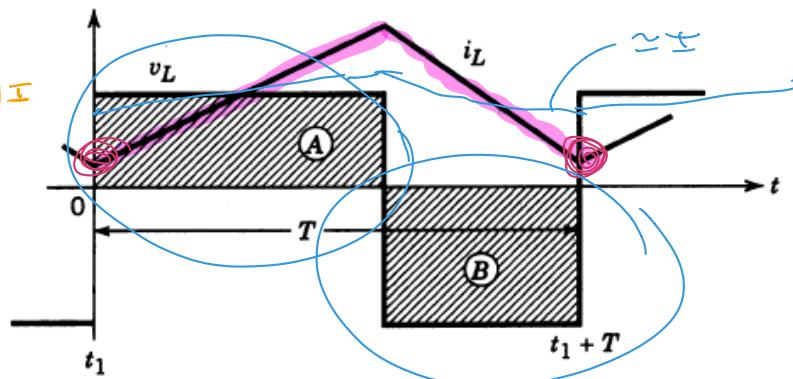
$v \neq 0 \text{ !!}$



$$\boxed{\bar{V}_L = 0}$$

$$v = L \frac{di}{dt}$$

$$\Delta i = \frac{v}{L} \Delta t$$





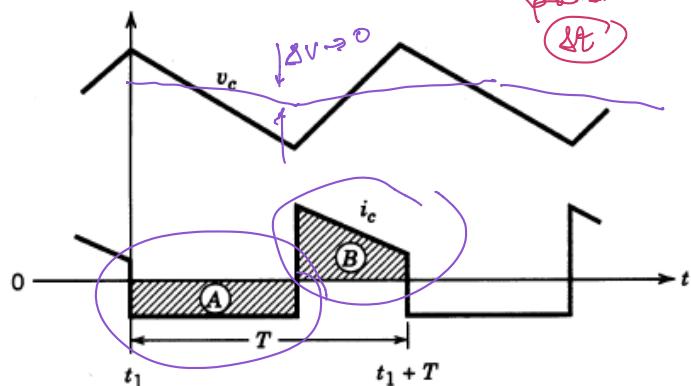
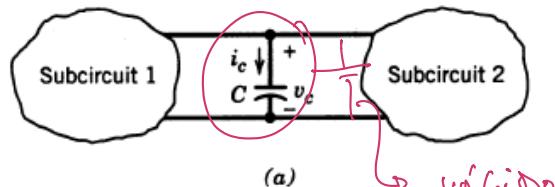
Condición de régimen permanente

Balance de carga en el condensador

$$i = C \frac{dv}{dt}$$

~~$i(0) = 0 \Rightarrow \Delta v \rightarrow 0 \Rightarrow \frac{v}{T} \neq 0$~~

~~$i(t) = 0$~~

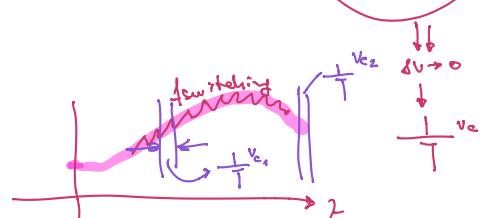


$$E = \frac{1}{2} CV^2$$

Rég. Pase:

$$\bar{V}_c = 0$$

$$i = C \frac{dv}{dt} \Rightarrow \Delta v = \frac{i}{C} \cdot \Delta t$$



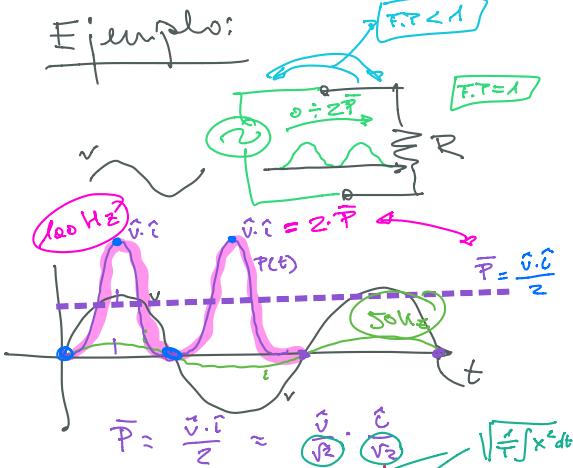


Valor medio y valor eficaz

Valor medio

$$\langle x \rangle = \frac{1}{T} \int_T x(t) dt$$

$$\begin{aligned} & V_{\text{se wt}} \\ & I_{\text{se wt}} \\ & S_{\text{se wt}} = \frac{V_{\text{se wt}} \cdot I_{\text{se wt}}}{2} \\ & V_{\text{E}} = \frac{V_{\text{se wt}} + V_{\text{se wt}}}{2} \end{aligned}$$



Valor eficaz

$$x_{ef} = \sqrt{\frac{1}{T} \int_T x^2 dt}$$

$\boxed{P(t) = V(t) \cdot i(t) = C(t) \cdot R}$

$$\boxed{\bar{P} = \frac{1}{T} \int_T P(t) dt = \frac{1}{T} \int_T i^2 dt = i_{ef}^2}$$

$\boxed{\bar{P} = V_{ef} \cdot I_{ef}}$ OJO!!

$\phi = 0$ $\boxed{S = V_{ef} \cdot I_{ef}}$ POTENCIA APARENTE

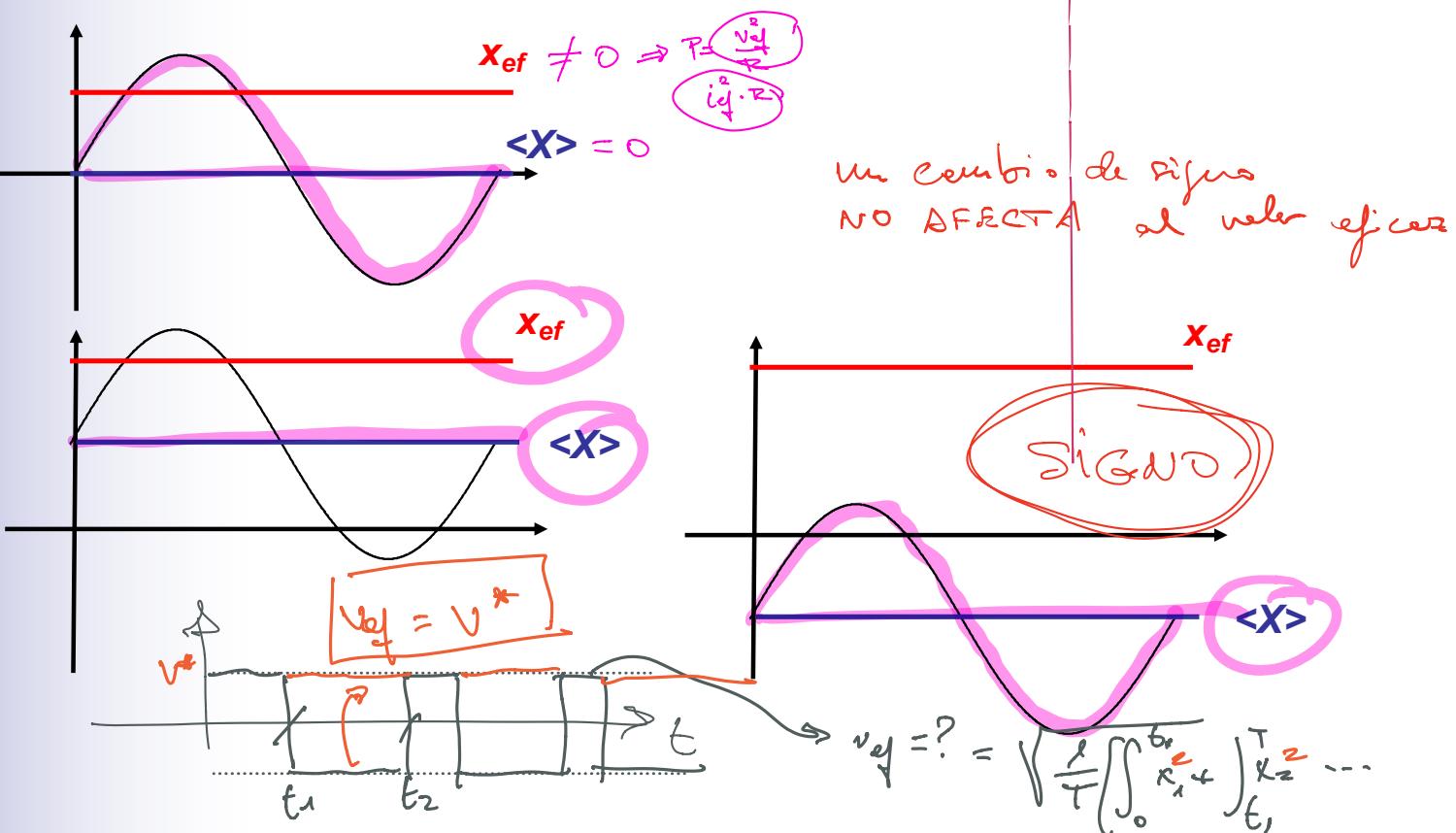
$\boxed{P = V_{ef} \cdot I_{ef} \cdot \cos \phi}$ PRIMER ARMÓNICO

$\boxed{P = \sum_{j \text{ ARMÓNICAS}} V_{ej} \cdot I_{ej} \cdot \cos \phi_j}$ FACTORES DE POTENCIA desfase

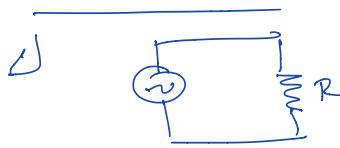


Valor medio y valor eficaz

Ejemplos



EJEMPLOS



$$\begin{aligned} P &< ? \\ \text{a) } P(t) &\rightarrow \hat{P} = \langle P \rangle = \frac{\hat{V} \cdot \hat{I}}{2} = \frac{\hat{V}^2}{2R} \xrightarrow{\text{cte!}} \text{señal armónica l.} \\ \text{b) } \hat{P} &= \cos \varphi \cdot \hat{P} = 1 \Rightarrow (\overline{P} = 1) \\ \text{c) } P_{\text{DT}} &= 0 \end{aligned}$$

Dissipación de Potencia innecesaria.

- 3) POTENCIA en una FUENTE de CORRIENTE:
- a) b)

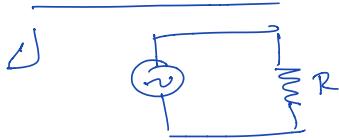
3) PÉRDIDAS en CONDUCCIÓN:

a) $P_{\text{cond-d}} = ?$

b) $P_{\text{cond-mos}} = ?$

$$\begin{aligned} P_{i_d} &= r_d \cdot i_d^2 \\ P_{V_B} &= V_B \cdot \bar{I} \\ P_{\text{cond}} &= P_{i_d} + P_{V_B} \\ P &= R_{\text{cond}} \cdot \bar{(i^2)} \end{aligned}$$

EJEMPLOS



$$\begin{aligned} P &< ? \\ \text{a) } P(t) & \\ \text{b) } \hat{P} & \\ \text{c) } P_{\text{DT}} & \end{aligned}$$

3) POTENCIA en una FUENTE de CORRIENTE

a) $\Rightarrow P(t)$

b) $\downarrow V$

$$P(t) = V_B \cdot i(t)$$

cte

$$\bar{P} = \frac{1}{T} \int V_B \cdot i dt$$

$$\boxed{\bar{P} = V_B \cdot \bar{i}}$$

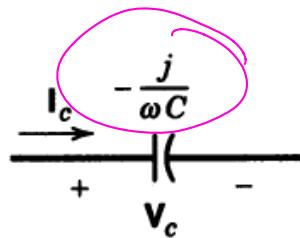
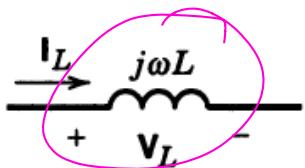
cte

$$P(t) = I_{\text{cc}} \cdot V(t)$$

$$\boxed{\bar{P} = I_{\text{cc}} \cdot \bar{V}}$$

cte

Régimen Sinusoidal Permanente

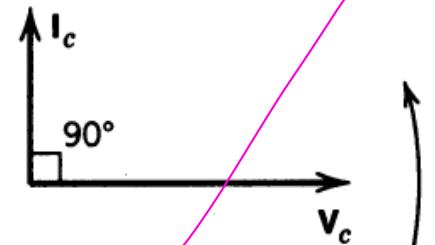
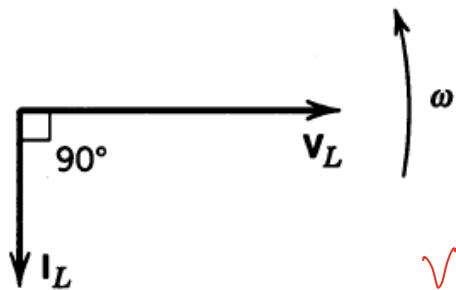


Caso Particular

$$v = L \frac{di}{dt}$$

$$i = C \frac{dv}{dt}$$

Siempre!!



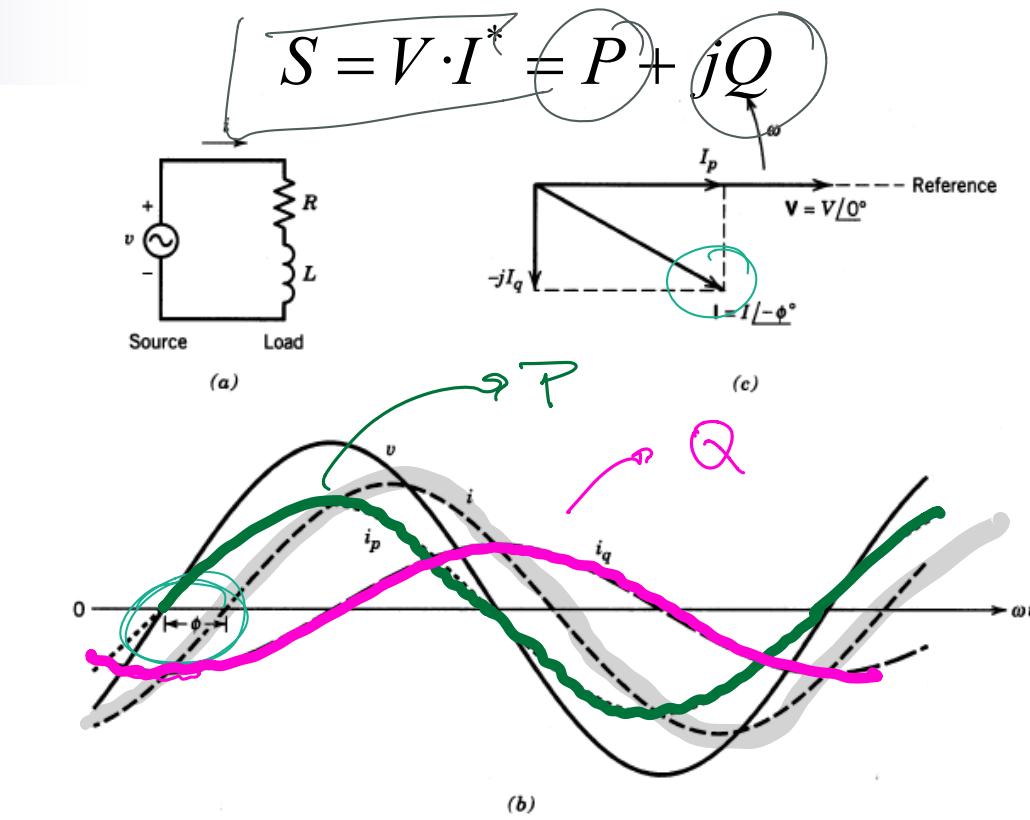
Graph of current squared over time. The area under the curve is shaded grey. The formula is given as $E = \frac{1}{2} L i^2$.

Graph of current i over time t . The current is sinusoidal. The formula is given as $V_{ref} = \sqrt{2}$.

Graph of current effective value i_{ef} over time t . The current is sinusoidal. The formula is given as $i_{ef} = \frac{C}{\sqrt{2}}$.

Régimen Sinusoidal Permanente

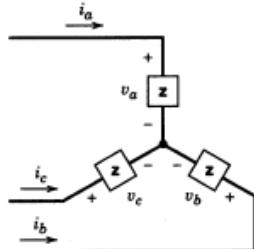
Potencia, Potencia aparente, potencia reactiva



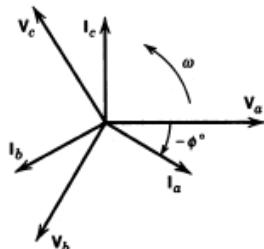
Trabajando con valores eficaces!!

Régimen Sinusoidal Permanente

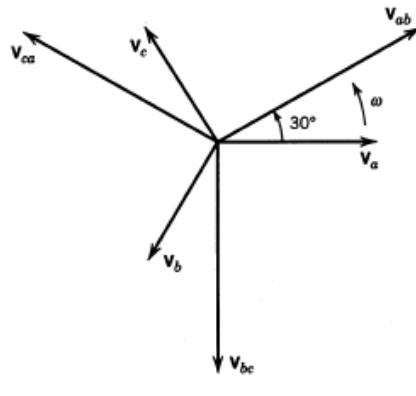
Circuitos trifásicos



(a)



(b)



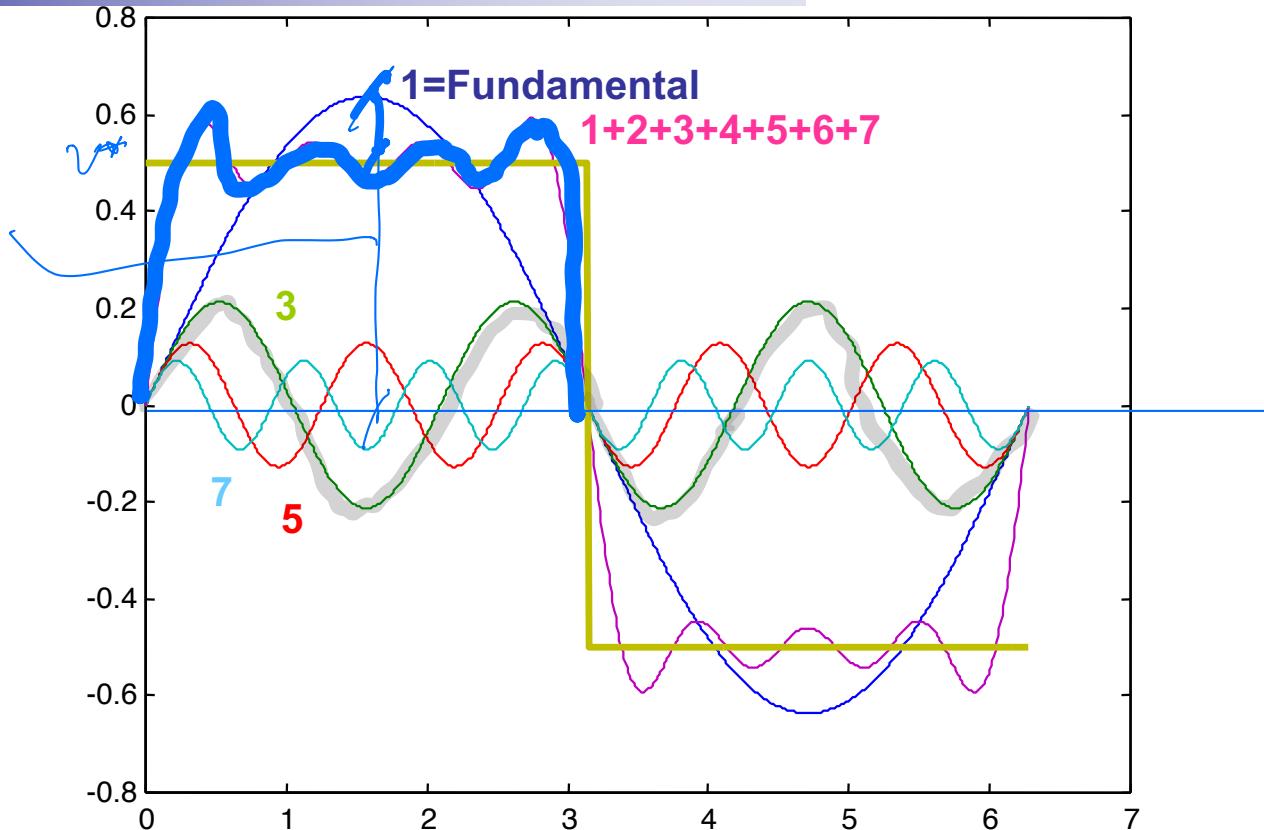
(c)

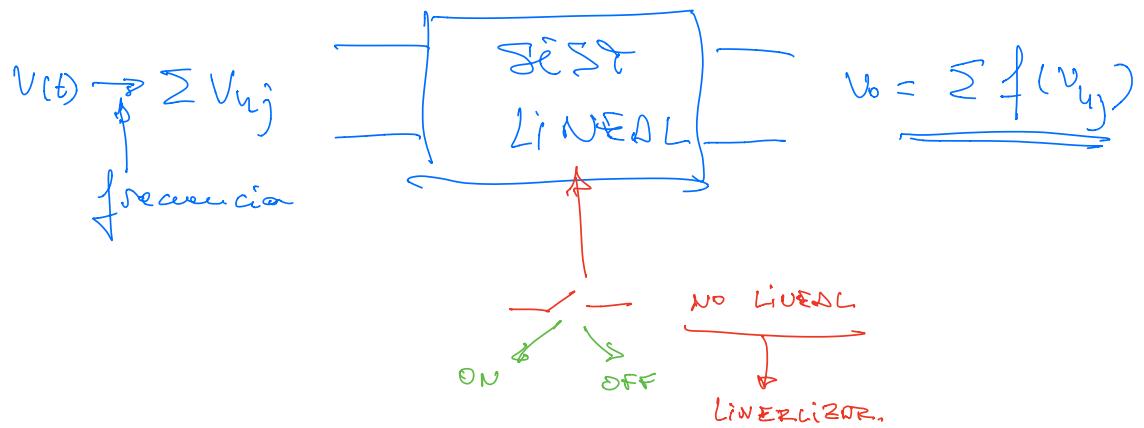
... pero en E. Potencia,
hay dispositivos conmutando,
que producen “armónicos”...

Descomposición en armónicos

Ejemplo

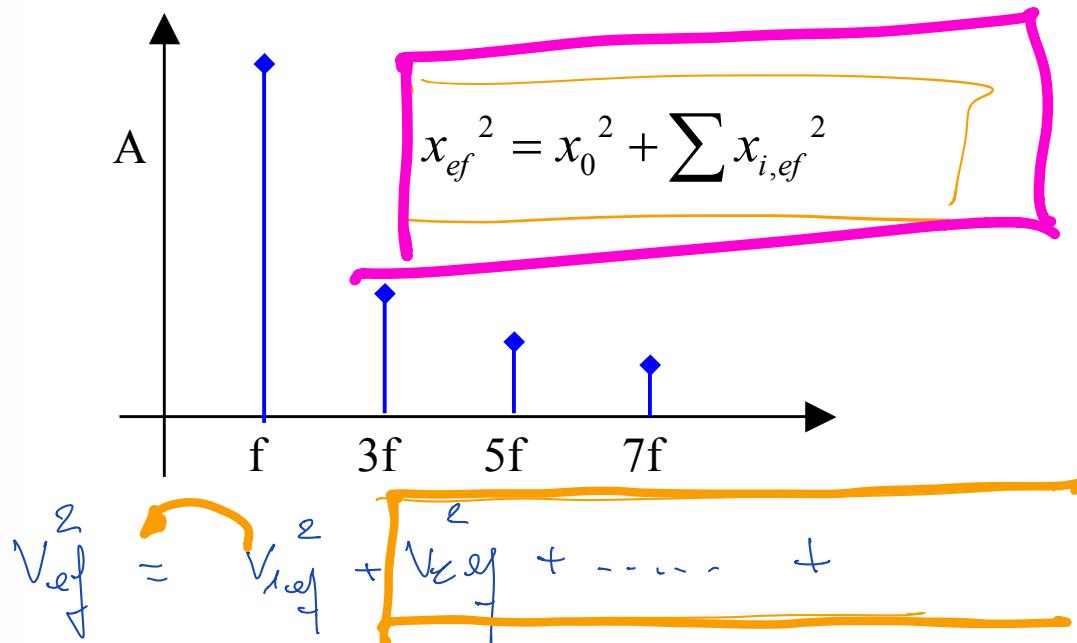
$$\frac{4}{\pi} V^*$$





Descomposición en armónicos

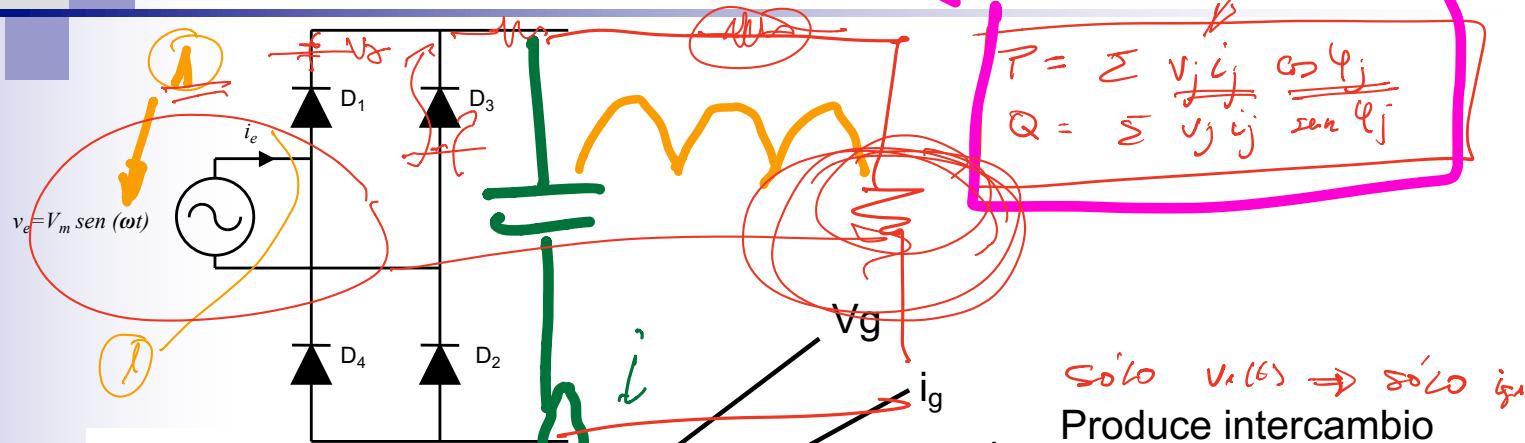
Ejemplo



El valor eficaz al cuadrado es la suma de los cuadrados de los valores eficaces



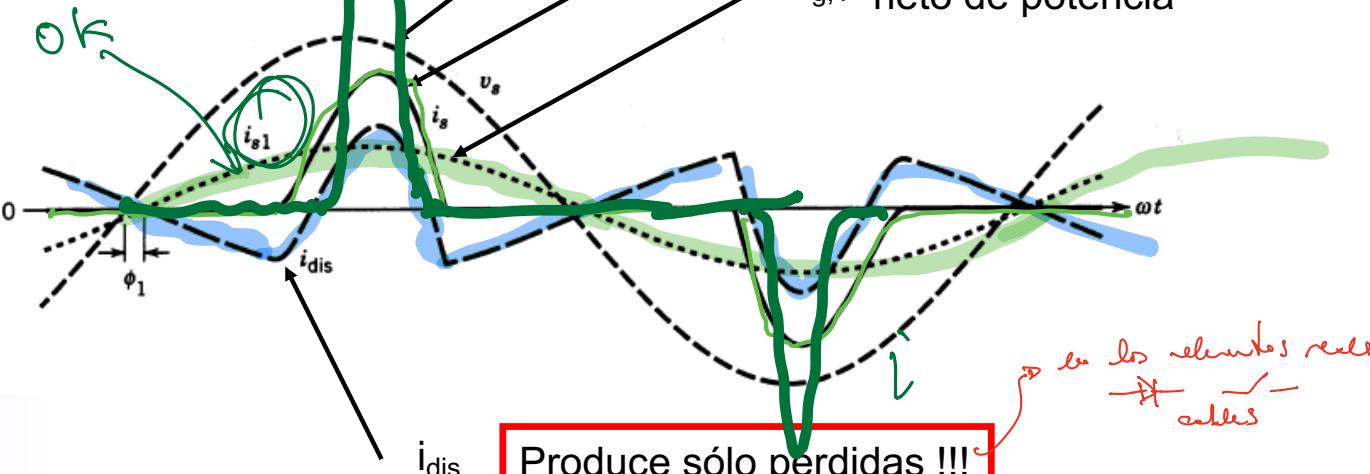
Distorsión en la corriente de línea



sólo $v_g(s)$ \Rightarrow sólo i_g

Produce intercambio neto de potencia

$i_{g,1}$

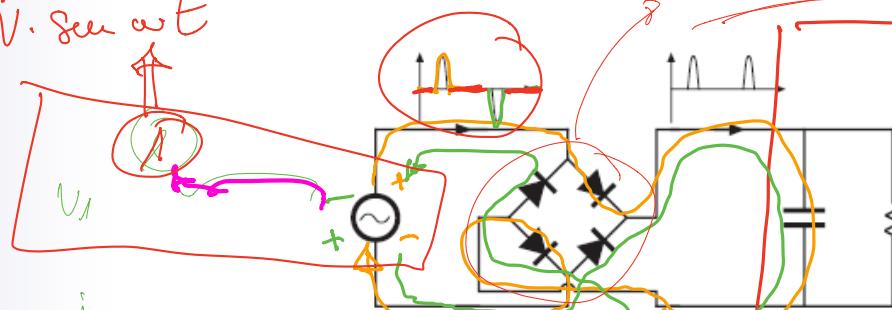


NO ENTRE GA P a la carga

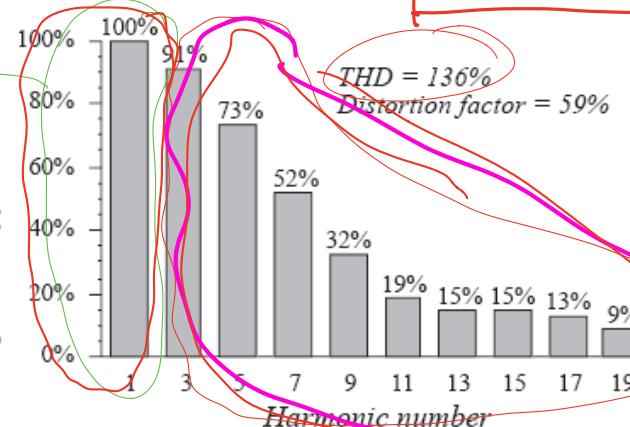
Distorsión en la corriente de línea



$\tilde{V} = \text{Sen } \omega t$



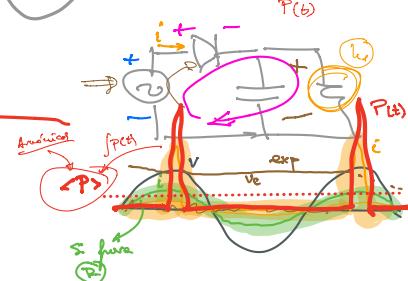
$$P = V_{ref} \cdot I_{ref} \cdot \cos \varphi$$



$$P = V_{ref} \cdot I_{ref} \cdot \cos \varphi_1 + V_{ref} \cdot I_{ref} \cdot \cos \varphi_2 + \dots$$

...

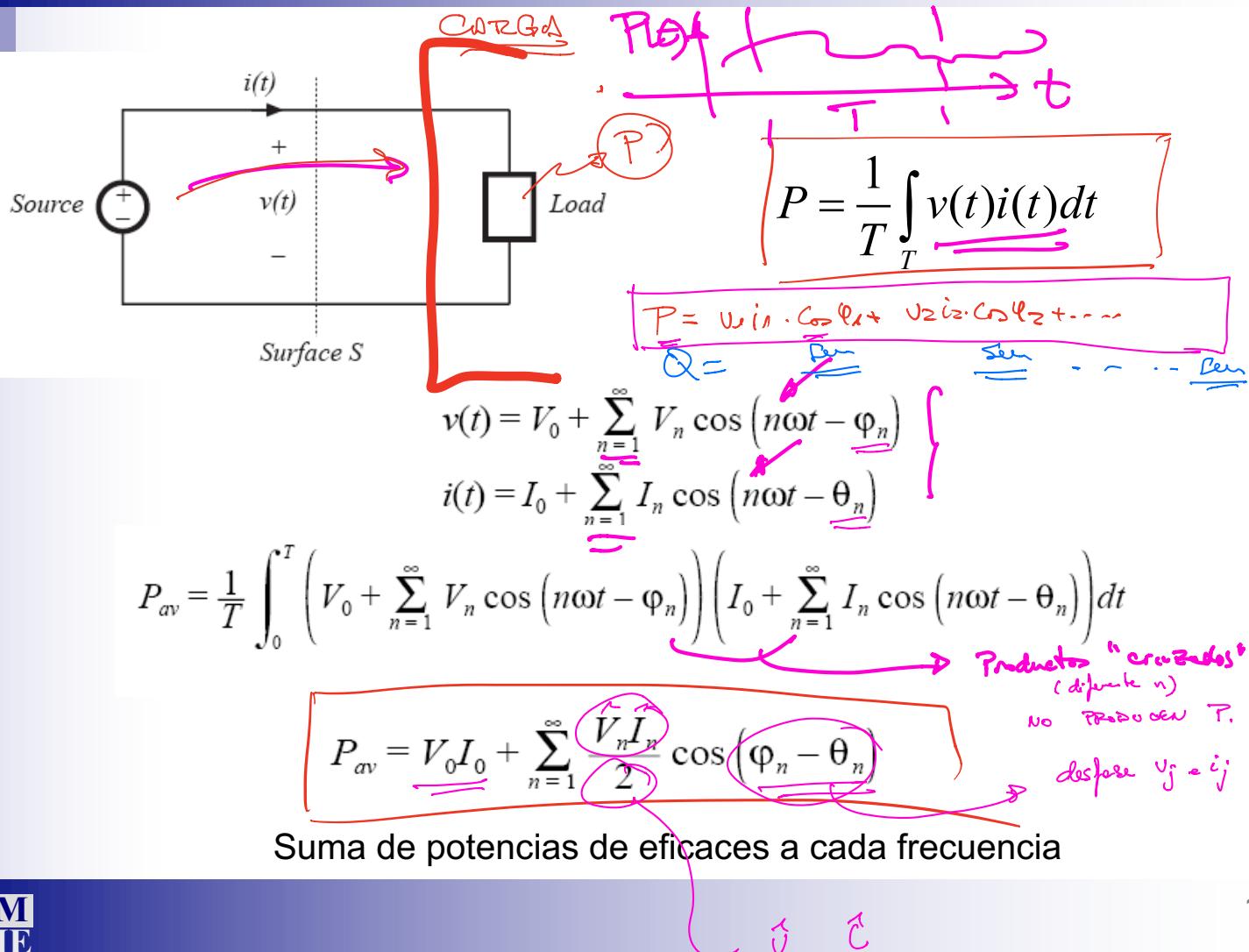
Enseguida vemos por qué es importante la THD



No entregan P a la carga.



Potencia media





Potencia media

$\sqrt{2}$, $\sqrt{2}$

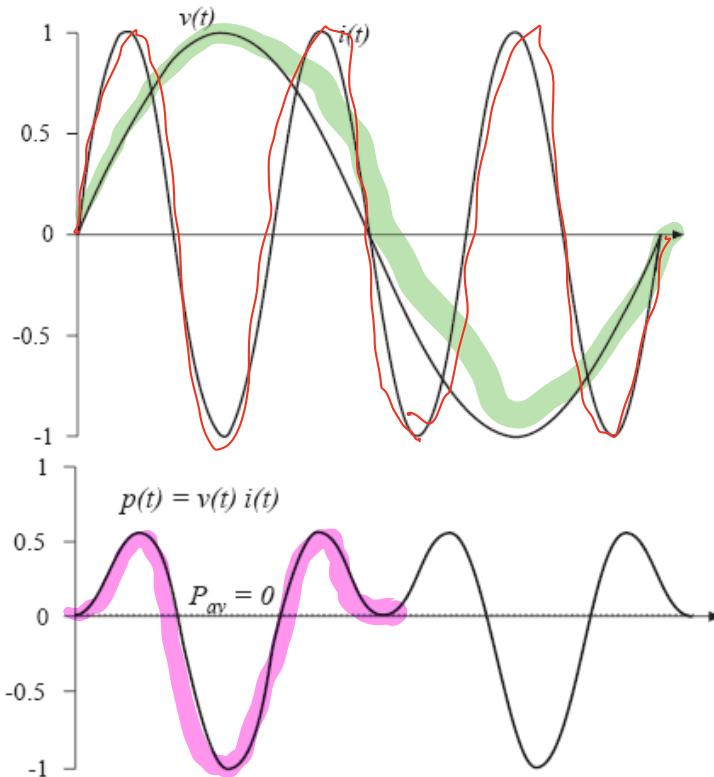
Ejemplo 1

Tensión fundamental

Corriente: 3er armónico

$$P=0$$

Un 3er armónico de i
desde la red de 50Hz
NO proporciona Pactiva !!





Potencia media

Ejemplo 2

Fourier series:

$$v(t) = 1.2 \cos(\omega t) + 0.33 \cos(3\omega t) + 0.2 \cos(5\omega t)$$

$$i(t) = 0.6 \cos(\omega t + 30^\circ) + 0.1 \cos(5\omega t + 45^\circ) + 0.1 \cos(7\omega t + 60^\circ)$$

Average power calculation:

$$P_{av} = \frac{(1.2)(0.6)}{2} \cos(30^\circ) + \frac{(0.2)(0.1)}{2} \cos(45^\circ) = 0.32$$



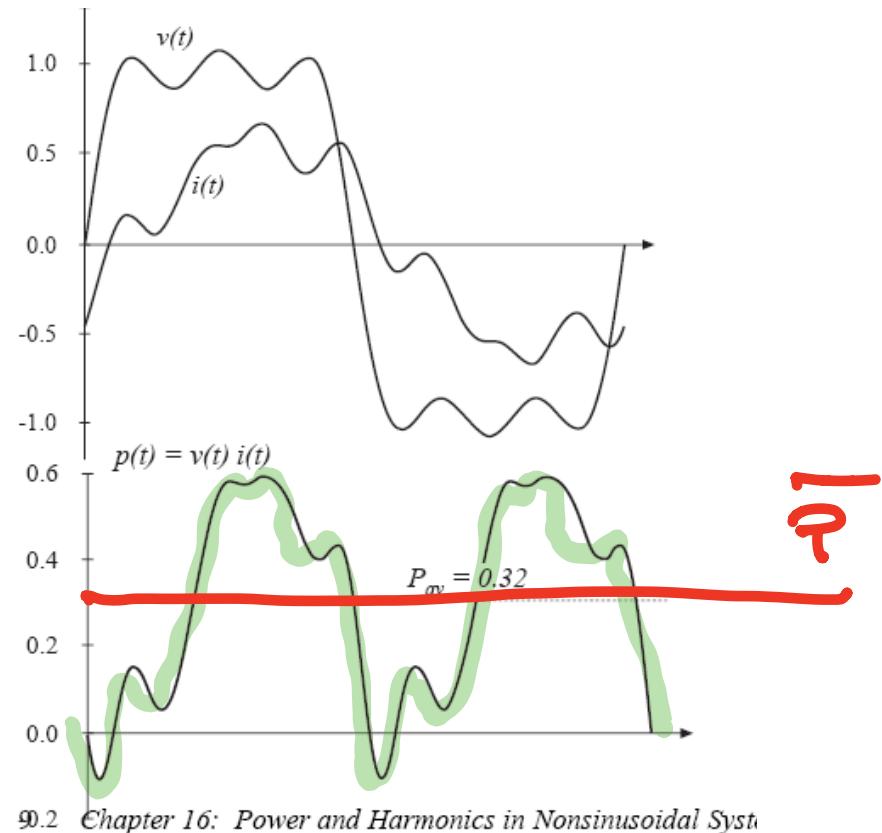
Potencia media

Ejemplo 2

Voltage: 1st, 3rd, 5th

Current: 1st, 5th, 7th

Power: net energy is transmitted at fundamental and fifth harmonic frequencies

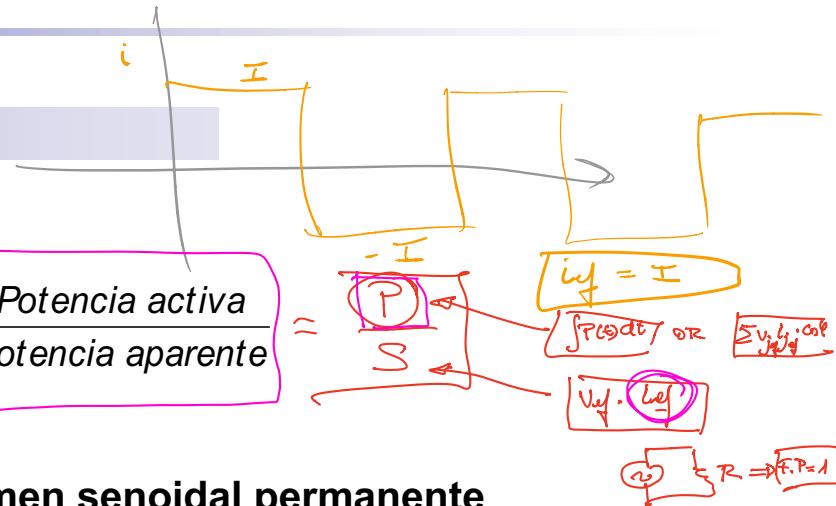




Definiciones (I)

Factor de Potencia

$$F.P. = \frac{\frac{1}{T} \int_0^T u \cdot i \cdot dt}{\sqrt{\frac{1}{T} \int_0^T i^2 \cdot dt} \sqrt{\frac{1}{T} \int_0^T u^2 \cdot dt}} = \frac{\text{Potencia activa}}{\text{Potencia aparente}}$$



F.P. = $\cos \theta$ sólo en régimen senoidal permanente



Distorsión armónica total

$$D.A.T. = \sqrt{lef_2^2 + lef_3^2 + \dots}$$

Asumimos
V_{ref}
So Hz

$$lef_1 = lef_1 + \frac{2}{2} i_{efj}$$



Si la tensión de entrada es senoidal

Factor de Potencia

$$F.P. = \frac{V_{ef} \cdot I_{ef1} \cdot \cos \theta}{V_{ef} \cdot I_{ef}} = \frac{I_{ef1}}{I_{ef}} \cos \theta = K_d \cdot K_\theta$$

$K_d = \frac{I_{ef1}}{I_{ef}}$ ⇒ factor de distorsión

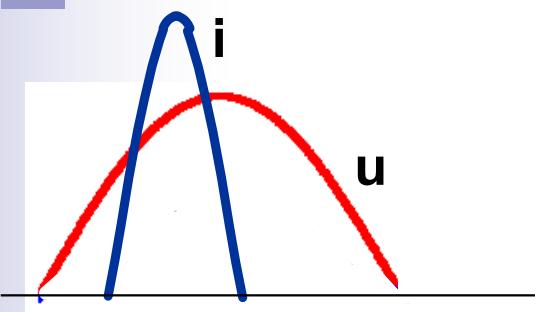
$K_\theta = \cos \theta$ ⇒ factor de desplazamiento

Relación entre FP y DAT

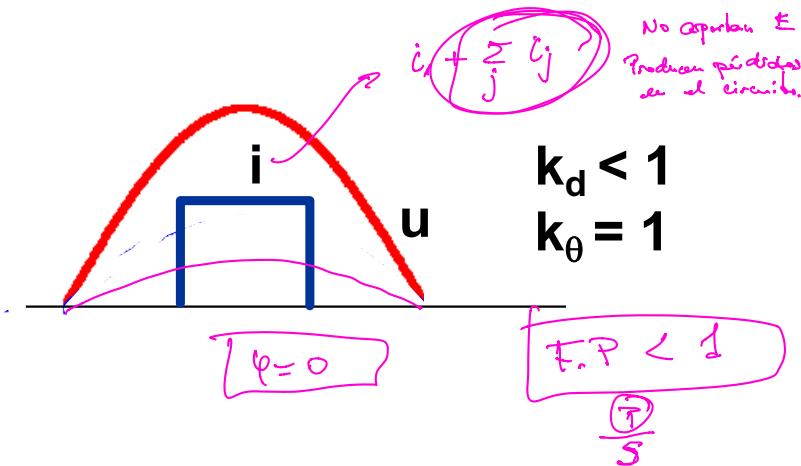
$$P.F. = \frac{\cos \theta}{\sqrt{1 + DAT^2}}$$



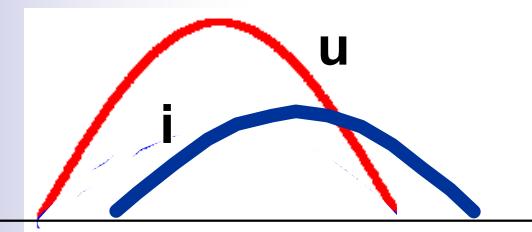
Ejemplos



$$\begin{aligned}k_d &< 1 \\k_\theta &< 1\end{aligned}$$

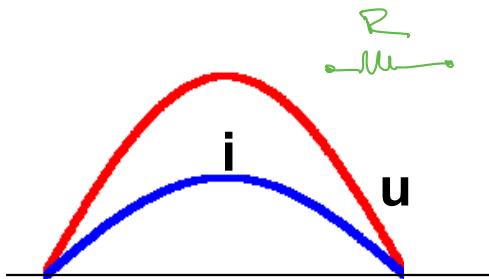


$$\begin{aligned}k_d &< 1 \\k_\theta &= 1\end{aligned}$$



$$\begin{aligned}k_d &= 1 \\k_\theta &< 1\end{aligned}$$

$$\begin{aligned}P &= V_{ref.} \cdot I_{ref.} \cdot \cos \varphi \\F.P. &= \frac{V_{ref.} \cdot I_{ref.} \cdot \cos \varphi}{V_{ref.} \cdot I_{ref.}} \\&= \cos \varphi\end{aligned}$$

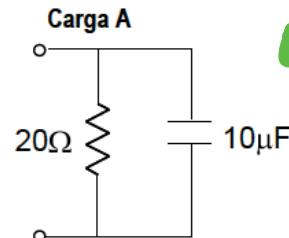
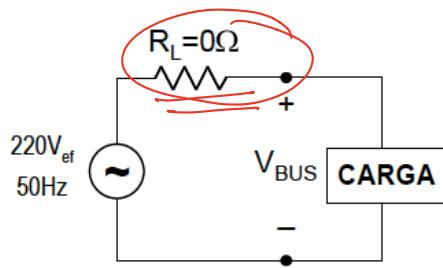


$$\begin{aligned}k_d &= 1 \\k_\theta &= 1\end{aligned}$$

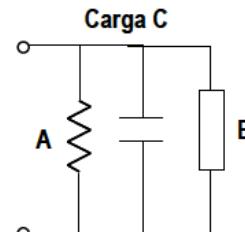
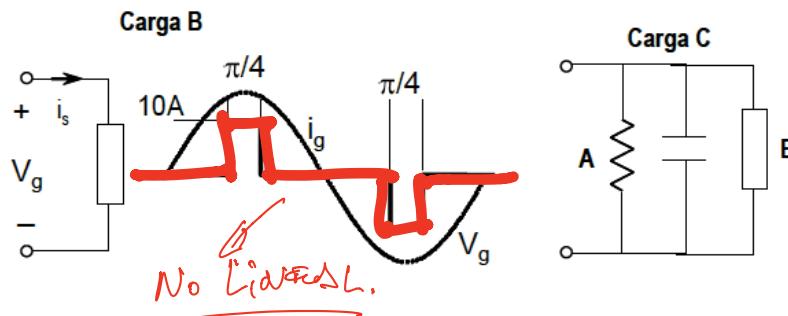
PROBLEMA 2. (3 puntos)

Para los siguientes tipos de carga alimentados desde un generador de tensión alterna ideal, determinar:

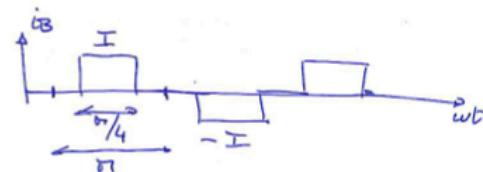
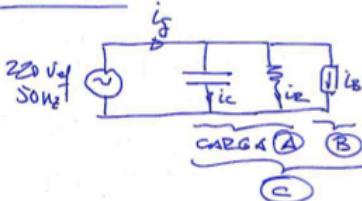
- Potencia aparente manejada por el generador.
- Potencia media consumida por la carga
- Factor de potencia y distorsión armónica total de corriente.
- Suponiendo que $R_L=0\Omega$, y para la carga C, determinar la distorsión armónica de tensión en V_{BUS} (asumir que la corriente por la carga no se ve afectada por R_L).



LíneaAL



PROBLEMAS



Fórmulas:

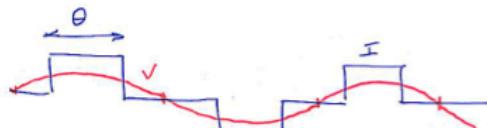
$$P = \frac{1}{T} \int_0^T V(t) \cdot i(t) dt = V_0 I_0 + V_{ref} L_{ref} \cdot \cos \varphi_0 + \dots + V_{ref} \cdot L_{ref} \cdot \cos \varphi_n$$

$$V_{ref}^2 = V_{ref1}^2 + V_{ref2}^2 + \dots \Rightarrow \sum_{n=1}^{\infty} V_{refn}^2 = V_{ref}^2 - V_{ref1}^2$$

$$F.P = \frac{P}{S}$$

$$\left| \begin{array}{l} S = P + jQ = V_{ref} \cdot I_{ref} \\ |S| = \sqrt{P^2 + Q^2} = V_{ref} \cdot I_{ref} \end{array} \right| \quad \left| \begin{array}{l} TWD = \frac{\sqrt{L_{ref1}^2 + C_{ref1}^2 + \dots}}{I_{ref1}} \\ = \sqrt{\frac{I_{ref1}^2}{I_{ref1}^2} - 1} \end{array} \right.$$

Cálculo del armónico n:



$$P = \frac{1}{T} \int_{-\pi/2}^{\pi/2} I \cdot V_p \cdot \cos wt dt = V_0 I_0 + V_{ref} L_{ref} \cdot \cos \varphi_0 + \dots + V_{ref} L_{ref} \cdot \cos \varphi_n$$

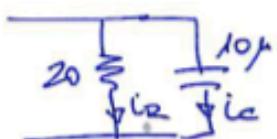
$$\frac{1}{T} \cdot I \cdot V_p \cdot \left[\sin \omega t \right]_{-\pi/2}^{\pi/2} = V_{ref} L_{ref} \cdot \cos \varphi_n = \frac{V_p}{\pi} \cdot L_{ref}$$

$$L_{refn} = I \cdot \frac{1}{\pi \cdot n} \cdot 2\sqrt{2} \cdot \sin \frac{n\theta}{2}$$

$$i_n(t) = I \cdot \frac{4}{\pi \cdot n} \cdot \sin \frac{n\theta}{2} \cdot \sin \omega t$$

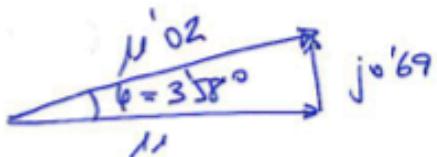
	i_{eq}	i_{eqj}	$\frac{1}{2} i_{eqj}^2$	P(w)	S(VA)	F.P	THD
Carga A	$11'02 \angle 358^\circ$	$i_{eq} = i_{eqj}$	0	$\frac{220^2}{R} = 2420$	$220 \cdot 11'02 = 2424$	$\frac{P}{\text{Ref}} = \frac{2424}{2424} = 0'998$	0
" B	5A	3'445	13'129	758	$220 \cdot 5 = 1100$	$\frac{758}{1100} = 0'69$	105 %.
" C	14'90	14'46	13'129	3178	$220 \cdot 14'9 = 3280$	$\frac{3178}{3280} = 0'97$	25 %.

Carga A :



$$i_{eqj} = \frac{220}{\sqrt{2}} = 11$$

$$i_{eqj} = \frac{220}{j\omega L} = 220 \cdot 2\pi \cdot 50 \cdot 10 \cdot 10^{-6} = j 0'69 \quad \left. \right\} 50 \text{ Hz}$$



Carga B :

$$\left\{ \begin{array}{l} i_{efD} = 10 \cdot \sqrt{\frac{\pi/4}{\pi}} = 5A \end{array} \right.$$

$$i_{efB_1} = \frac{\pi}{\pi} \cdot 2 \cdot \sqrt{2} \cdot \sin \frac{\pi}{8} = 3'445A$$

$$P = \frac{1}{\pi} \int_{-\pi/8}^{\pi/8} I \cdot V_p \cdot \cos \omega t d\omega t = \frac{I \cdot V_p}{\pi} \cdot 2 \cdot \sin \frac{\pi}{8} = \frac{10 \cdot 220 \cdot \sqrt{2} \cdot 2}{\pi} \cdot \sin \frac{\pi}{8}$$

$$F.P = \frac{758}{220 \cdot 5}$$

$$= 0'69 = 69\%$$

$$\left(\sum_{m=2}^{\infty} \frac{i_{efm}^2}{V_m} \right) = 5^2 - 3'445^2 = 13'129$$

$$THD = \frac{\sqrt{13'129}}{3'445} = 1'05$$

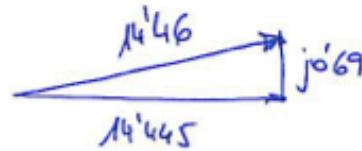
Carga C:

$$P_c = P_d + P_B = 758 + 2420 = 3178W$$

$$\sum_{m=2}^{\infty} \frac{i_{efm}^2}{V_m} = 13'129, \text{ igual que en B}$$

Primer armónico:

$$\begin{aligned} i_{efc} &= i_{efc1} + \sum_{m=2}^{\infty} i_{efm}^2 \\ &+ j0'69 \end{aligned}$$

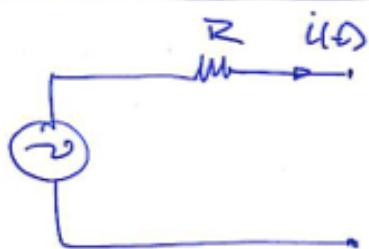


$$i_{efc}^2 = i_{efc1}^2 + \sum_{m=2}^{\infty} i_{efm}^2 = 14'46^2 + 13'129 = 222 \Rightarrow i_{efc} = 14'9$$

$$F.P = \frac{P}{S} = \frac{3178}{220 \cdot 14'9} = 0'97$$

$$THD = \frac{\sqrt{13'129}}{14'46} = 0'25$$

THD de v , conocida $\sum_{n=2}^{\infty} i_{ef,n}^2$:

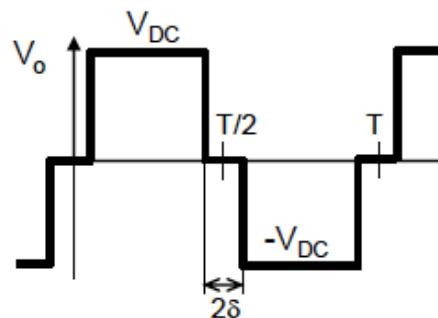


$$\boxed{THD = \frac{\sqrt{\sum i_{ef,n}^2 \cdot R}}{V_{ef,r}} = \frac{R}{V_{ef,r}} \cdot \sqrt{\sum_{n=2}^{\infty} i_{ef,n}^2}}$$

$$\boxed{THD = \frac{0'1}{220} \cdot \sqrt{13'129} = 0'16\%}$$

PROBLEMA 3. (2,5 ptos)

La descomposición en serie de Fourier de la forma de onda de la Figura 1 es la que se muestra a su derecha.



$$v_o(t) = V_{DC} \sum_{n=1,3,5} \frac{4}{n\pi} \cos(n\delta) \sin(n\omega t)$$

Figura 1

El circuito de la figura 2 representa un inversor no modulado que puede inyectar y absorber potencia de la red eléctrica. El inversor genera una tensión desfasada de un ángulo Φ respecto a la tensión de red, tal y como se muestra en la figura 3.

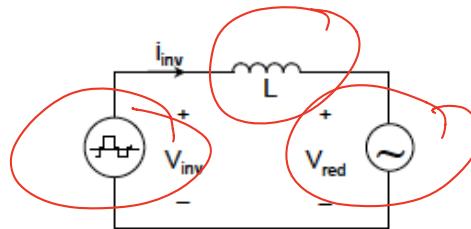


Figura 2

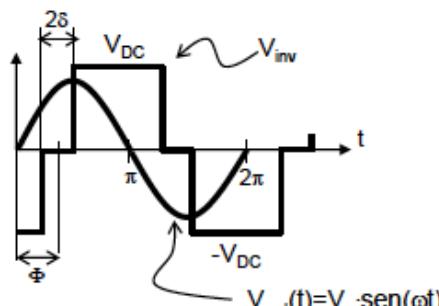


Figura 3

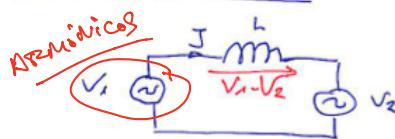
Datos:

$L=2\text{mH}$
 $\delta=37^\circ$
 $\Phi=10^\circ$
 $V_p=300\text{V}$
 $V_{DC}=300\text{V}$
 $f=50\text{Hz}$

Calcular:

- Amplitud de los armónicos 1, 3 y 5 de corriente (valor de pico).
- Potencia inyectada a la red por los armónicos 1, 3 y 5 de corriente (signo positivo absorbida por la red).
- Potencia que el inversor inyecta debida a los armónicos 1, 3 y 5.
- Factor de potencia en el lado de la red y factor de potencia en el lado del inversor (hasta 5º armónico).
- Distorsión armónica total de la corriente (hasta 5º armónico).

FLUJO DE POTENCIA



$$J = \frac{V_1 - V_2}{j\omega L}$$

$$S_1 = V_1 \cdot J^* = V_1 \cdot \frac{V_1^* - V_2^*}{-j\omega L} = -\frac{V_1^2}{j\omega L} + \frac{V_1 \cdot V_2^*}{j\omega L}$$

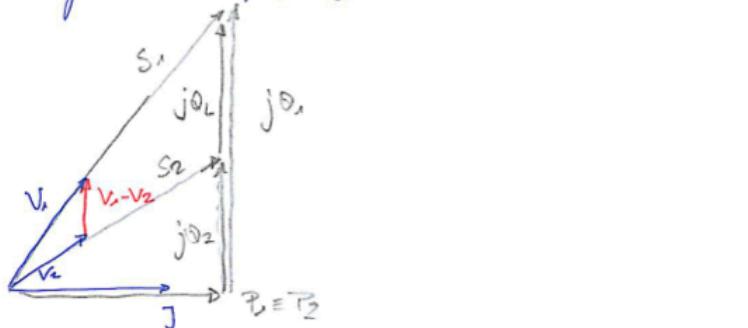
$$P_1 = P_2 = \operatorname{Re} \left\{ \frac{V_1 \cdot V_2^*}{j\omega L} \right\} = \frac{V_1 \cdot V_2}{\omega L} \cdot \cos(\varphi_1 - \varphi_2 - 90^\circ)$$

$$\boxed{P_1 = \frac{V_1 \cdot V_2}{\omega L} \cdot \sin(\varphi_1 - \varphi_2)} = P_2$$

$$Q_1 = \operatorname{Im} \left\{ j \frac{V_1^2}{\omega L} + \frac{V_1 \cdot V_2^*}{j\omega L} \right\}$$

$$\boxed{Q_1 = \frac{V_1^2}{\omega L} + \frac{V_1 \cdot V_2}{\omega L} \cdot \sin(\varphi_1 - \varphi_2 - 90^\circ) = \frac{V_1^2}{\omega L} + \frac{V_1 \cdot V_2}{\omega L} \cdot \cos(\varphi_1 - \varphi_2)}$$

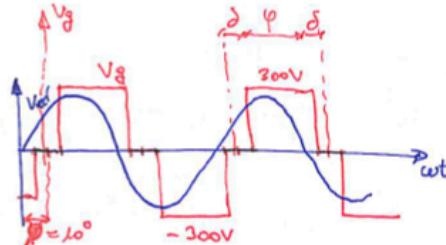
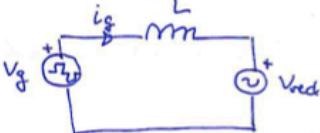
Con un desfase tan grande las dos fuentes aportan reactiva a la L. Con un desfase menor, sería:





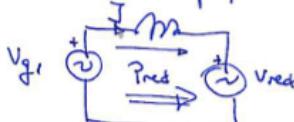
La P va de la V más adelantada hacia la retrasada, por lo que sería mejor resolverlo con el desfase al revés (v_g inversor adelantada respecto a v_{red})

PROBLEMA



$$\left. \begin{array}{l} \text{origen de} \\ \text{tension de} \\ \text{wt} = 10^\circ \end{array} \right\} \begin{aligned} V_g &= \left[\frac{4}{\pi} \cdot \cos \delta \cdot \sin \omega t + \frac{4}{3\pi} \cdot \cos 3\delta \cdot \sin 3\omega t + \frac{4}{5\pi} \cdot \cos 5\delta \cdot \sin 5\omega t \dots \right] \cdot 300 \\ &= \frac{4}{\pi} \cdot \sin \frac{\delta}{2} \cdot \sin \omega t + \frac{4}{3\pi} \cdot \sin \frac{3\delta}{2} \cdot \sin 3\omega t + \frac{4}{5\pi} \cdot \sin \frac{5\delta}{2} \cdot \sin 5\omega t \dots] \cdot 300 \\ \text{siendo } \delta &= \pi - 2\phi \Leftrightarrow \delta = \frac{\pi - \phi}{2} \Leftrightarrow \cos \delta = \sin \frac{\phi}{2} \end{aligned}$$

Aplicamos superposición para cada armónico:



$$V_{g1} = \frac{4}{\pi} \cos \delta \cdot 300 \cdot \frac{1}{\sqrt{2}} \\ = 215'7 \text{ V}$$

$$V_{g1} = 215'7 \cdot \sqrt{2} = 305 \text{ V}$$



$$I_{ref3} = \frac{V_{g3} \omega}{3\pi L} \\ = \frac{4}{3\pi} \cos 3\delta \cdot 300 \\ \frac{1}{\sqrt{2} \cdot 3 \cdot 100\pi \cdot 2 \cdot 10^{-3}}$$

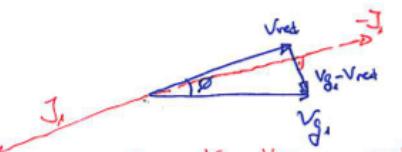
$$P_3 = 0$$

$$I_{ref5} = \frac{V_{g5} \omega}{5\pi L} \\ = \frac{4}{5\pi} \cos 5\delta \cdot 300 \\ \frac{1}{\sqrt{2} \cdot 5 \cdot 100\pi \cdot 2 \cdot 10^{-3}}$$

$$= \frac{2\sqrt{2} \cdot 0.5\delta \cdot 300}{5 \cdot \pi^2 \cdot 100 \cdot 2 \cdot 10^{-3}} \\ = 174$$

$$P_5 = 0$$

$$I_{ref5} = \sqrt{2} I_{ref3} = 24'2$$



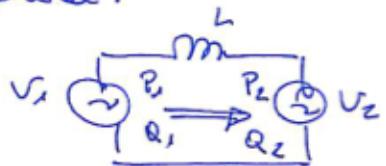
$$J_1 = \frac{V_{g1} - V_{red}}{j\omega L} = \frac{215'7 - \frac{300}{\sqrt{2}} \cdot e^{j10^\circ}}{j20\pi \cdot 2 \cdot 10^{-3}} = -38'6 - j \cdot 10'8 \Rightarrow I_{ref1} = 59'6$$

$$S_{ref1} = V_{g1} J_1^* = V_{g1} \left(\frac{V_{g1} - V_{red}}{j\omega L} \right)^* = V_{red} \cdot \frac{V_{g1}^* - V_{red}^*}{-j\omega L}$$

$$S_{red} = \frac{V_{red} \cdot V_{g1}^*}{-j\omega L} + \frac{V_{red}^2}{j\omega L}$$

$$P_{ref1} = \frac{V_{red} \cdot V_{g1}}{\omega L} \cdot \cos(\phi + 90^\circ) = -\frac{300}{\sqrt{2}} \cdot 215'7 \cdot \sin 10^\circ = -12645 \text{ W}$$

in general:



$$\boxed{P_2 = \frac{V_1 \cdot V_2}{\omega L} \cdot \sin(\varphi_1 - \varphi_2) = P_1}$$

$$Q_2 = \frac{V_1 \cdot V_2}{\omega L} \cdot \cos(\varphi_2 - \varphi_1) + \frac{V_2^2}{\omega L} \neq Q_1$$

d) Factor de Potencia

$$\underline{I_{ef}}^2 = \underline{I_{ef1}}^2 + \underline{I_{ef3}}^2 + \underline{I_{efS}}^2 \Rightarrow \underline{I_{ef}} = \sqrt{59'6^2 + 17'1^2 + 17'1^2} = 64'3$$

$$F.P_{red} = \frac{P}{V_{ef} \cdot I_{ef}} = \frac{12645}{\frac{300}{\sqrt{2}} \cdot 64'3} = 0'927$$

$$F.P_g = \frac{P}{V_{efg} \cdot I_{ef}} = \frac{12645}{300 \cdot \sqrt{1-\frac{25}{31}}} \cdot 64'3 = \frac{12645}{300 \cdot 0'767 \cdot 64'3} = 0'85$$

$$e) THD = \frac{\sqrt{\underline{I_{ef3}}^2 + \underline{I_{efS}}^2}}{\underline{I_{ef1}}} = \frac{\sqrt{17'1^2 + 17'1^2}}{59'6} = 0'4 = 40\%$$